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Transportation Research Procedia 3 (2014) 895 – 904

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17th Meeting of the EURO Working Group on Transportation, EWGT2014, 2-4 July 2014,  
Sevilla, Spain

## Dynamic assignment with user information in multimodal networks

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### Abstract

The dynamic assignment of users has been widely studied for the road network while it is less considered for multimodal networks. In this article, we investigate the dynamic assignment of users in multimodal transportation systems while differentiating between informed and uninformed users. The problem is modeled as a multiagent system where we consider all of the modes that share the road infrastructure (private vehicles, taxis, buses, tramway, electric car sharing services), thus the users and vehicles of each mode are represented by an agent.

As mentioned in the above, we consider two types of users: informed and uninformed. Our objective is to assess the impact of the presence of informed users on the dynamic assignment on the network. To do so, we provide an analytical study on the Braess paradox where we explore the possibility of improving the assignment of users through the information that is provided to them. The simulation model is cellular automata based and was executed on a multilevel network that includes a Braess paradox in order to validate the analytical results.

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Selection and peer-review under responsibility of the Scientific Committee of EWGT2014

**Keywords:** assignment; multiagent systems; system equilibrium; multimodal; simulation, information;

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### 1. Introduction

The dynamic assignment of users in a multimodal network is a complex problem that has been widely studied over the past decade. For instance, we cite the work of (Horn, 2002) who was interested in planning passenger's journeys in a multimodal network. (Hamdouche & Lawphongpanich, 2007) and (Hamdouche & al, 2004) worked on

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a space-time graph then used a dynamic propagative algorithm to find a strategy of minimum cost. In (Lo & al, 2001), authors chose a state augmenting procedure to deal with the multimodal network where they used a stochastic loading procedure to find the best assignment.

The principle of path viability has been studied by (Lozano & Storchi, 1999) where they introduced an algorithm that looks for a path that ameliorates the travel cost while introducing an additional transfer.

Many studies have also been carried out to construct guidance systems in multimodal networks where the users equipped with information systems are provided with multimodal itineraries based on real time information during their journey. (Kamoun, 2007) and (Zidi, 2006) used a multiagent architecture to create a guidance system that assigns the travelers to a multimodal itinerary of minimal cost. Introducing guidance brings an important question: How to calculate an assignment scheme that keeps the system in equilibrium without penalizing the users? For this issue, (Ma & Lebacque, 2012) calculate the system optimal predictive dynamic assignment using a multiagent simulation based model combined with a cross entropy algorithm.

The purpose of our paper is to evaluate the importance of the information that we give to the travelers. To do so we will go through a comparative study using a multiagent simulation based model where we represent two types of users: users equipped with information devices and unequipped users. We inject the two categories of users under different proportions in the network then we calculate the multimodal dynamic assignment and evaluate the state of the network.

The paper is organized as follow:

Section 1 represents the problem formulation and a theoretical study on the Braess paradox is given in section 2 to evaluate whether or not the equilibrium can be improved when guidance is given to the travelers. This analytical study will be validated using a multiagent simulation on a graph containing a Braess paradox. The description of the multiagent architecture and the computational study along with the numerical results are given in sections 3 and 4 respectively.

## 2. Problem formulation

The network we consider is modeled as a multilevel structure where each level represents a modal sub-network. Let  $G=(V,E)$  be a directed graph that represents the multimodal network, where  $V$  is the set of nodes and  $E$  the set of arcs. We refer to each sub network with  $G_m=(N_m, A_m)$ ,  $m \in M$ ,  $N_m$  is the set of nodes of mode  $m$  and  $A_m$  the set of arcs of mode  $m$ . We connect the nodes of different modes with walking arcs under the condition that the distance between the two nodes doesn't exceed a defined value  $\beta$ . Note that the multimodal network we consider contains a Braess paradox structure.

We assign to each arc a length  $L_{ij}$ , an average speed  $V_{ij}$  and a maximum speed  $V^{\max}$ .

In our formulation, we consider three types of modes: The vehicular (v), the metro (m) and the bus modes (b). The travel time  $c_{ij}(t)$  along the arc  $(i,j)$  of a given mode  $m$  is calculated by our simulation model. We assign to each vehicle a maximum capacity  $C$ .

We consider two types of users: equipped and unequipped. The equipped users are connected to a central entity and are assumed to follow the assigned route. The unequipped ones, they are supposed to choose their routes according to their knowledge of the network.

## 3. Importance of the information

We consider a multimodal network which is composed of two types of modes: a private transportation mode (metro) and a road network. The two networks are related with walking arcs that allows users to move from one mode to another as shown in the following figure. Let  $D$  be the demand that we have to split between the four paths

$$D = d_1 + d_2 + d_3 + d_4$$

We consider that the bus has three stop, two at the extremities and one at the modal exchange pole.

The costs of the arcs of the vehicular mode is given by the following equation

$$c_{ij}(d_{ij}) = \alpha L + v_{ij} \frac{L}{V_{ij}(d_{ij})}, (i, j) \in A_1 \quad (1)$$

$\alpha = 0.33$  represents the average cost per kilometer.

$L = 12km$  is the length of the path

$v_{ij} = 23euros / hr$  is the value of time for the users of the vehicular mode.

$V_{ij}(d_{ij})$  the average speed on arc (i,j)

We calculate the expression of the speed using the fundamental diagram.

$$V(d_{ij}) = V_{max} \left[ \frac{1}{2} + \sqrt{\frac{1}{4} - \frac{d_{ij}}{Q_{max}} \eta (1 - \eta)} \right] \quad (2)$$

$V_{max} = 55km / hr$  is the desired speed

$Q_{max} = 1100vh / hr$  is the maximum flow.

$$\eta = \frac{V_{crit}}{V_{max}} \text{ such as } V_{crit} = \frac{Q_{max}}{K_{crit}}$$

The expression of the costs of the arcs for the public transportation mode is given by the following expression

$$C_{ij}(d_{ij}) = \gamma + v_{ij} \tau + S v_{ij} \tau \frac{\text{Max}(0, d_{ij} - d_s)}{d_s} \quad (3)$$

$\gamma = 1.2euros$  is the monetary cost

$\tau = 0.53hr$  is the cost of the travel time which is evaluated using the value of the time  $v_{ij} = 15euros / hr$ .

$S = 1.6$  is a coefficient

$d_s = 800$  travelers/hr is the threshold of congestion

#### 4. Braess paradox and user information

In this section we study the importance of the information given to the travelers in a multimodal network using a case study which is the Braess paradox.

##### 4.1. Setting the scene

Let us consider the following graph

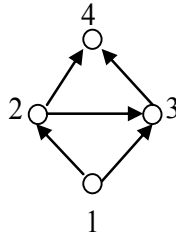


Figure 1. Braess paradox graph

Let  $D$  be the total demand on the graph given in Figure 1,  $d_{ij}^a$  be the demand on arc  $(i, j)$  and  $d_k^p$  the demand on path  $k$ .

As shown in figure 1, we consider three paths:  $P_1 = \{(1, 2), (2, 4)\}$ ,  $P_2 = \{(1, 2), (2, 3), (3, 4)\}$  and  $P_3 = \{(1, 3), (3, 4)\}$ . Here, we consider that  $P_1$  is a public transportation path,  $P_3$  a vehicular path and  $P_2$  a multimodal path.

The demand on each arc is given in the following equation

$$d_{12}^a = d_1^p + d_2^p, d_{23}^a = d_2^p, d_{13}^a = d_3^p, d_{24}^a = d_1^p, d_{34}^a = d_2^p + d_3^p \quad (4)$$

Let  $c_{ij}$  be the cost of arc  $(i, j)$ ,  $\forall i, j \in \{1, \dots, 4\}$ , where  $c_{12} = \alpha + \beta d_{12}^a$ ,  $c_{13} = \gamma + \delta d_{13}^a$ ,  $c_{23} = \lambda + \mu d_{23}^a$ ,  $c_{24} = \gamma + \delta d_{24}^a$  and  $c_{34} = \lambda + \mu d_{34}^a$ .

Which gives us the cost of each path  $c_k^p$ ,  $k = \{1, 2, 3\}$  as the following

$$\begin{aligned} c_1^p &= \alpha + \beta(d_1^p + d_2^p) + \gamma + \delta d_1^p \\ c_2^p &= \alpha + \beta(d_2^p + d_3^p) + \gamma + \delta d_3^p \\ c_3^p &= \alpha + \beta(d_1^p + d_2^p) + \lambda + \mu d_2^p + \alpha + \beta(d_2^p + d_3^p) \end{aligned} \quad (5)$$

Assuming symmetry between paths 1 and 3, the demands on those paths are equal, which gives us the following equation

$$d_1 = d_3 = \frac{D - d_2}{2} \quad (6)$$

Thus we rewrite the cost on each path as follows

$$\begin{aligned} c_1^p = c_3^p &= \alpha + \gamma + \frac{\beta + \delta}{2} D + \frac{\beta - \delta}{2} d_2^p \\ c_2^p &= 2\alpha + \lambda + \beta D + (\beta + \mu) d_2^p \end{aligned} \quad (7)$$

We assume that  $\beta \gg \delta$  because links (1,2) and (3,4) are much more sensitive to demand and  $\gamma > \alpha + \lambda$

#### 4.2. User equilibrium with no information

We calculate the user equilibrium where we consider only unequipped users and we obtain three cases: the case where only path 2 is used, the case where only paths 1 and 3 are used and finally the case where all the paths are used. We will treat each case in the following

- Case 1: Only path 2 is used

$$d_2^p = D \text{ and } d_1^p = d_3^p = 0$$

$$c_2^p < c_1^p = c_3^p \Rightarrow D \leq \frac{\gamma - (\alpha + \lambda)}{\beta + \mu}$$

The corresponding OD cost is then

$$S = c_2^p = 2\alpha + \lambda + (\mu + 2\beta)D \quad (8)$$

- Case2: Paths 1 and 3 are used

$$d_2^p = 0, d_1^p = d_3^p = \frac{D}{2}$$

$$c_1^p = c_3^p < c_2^p \Rightarrow D \geq \frac{\gamma - (\alpha + \lambda)}{\beta - \delta} \quad (9)$$

The corresponding OD cost is

$$S = c_1^p = c_3^p = \alpha + \gamma + \frac{\beta + \delta}{2} D$$

- Case3: Paths 1,2 and 3 are used which means that  $c_1^p = c_2^p = c_3^p, d_2^p \in [0, D]$

Thus, the demand on the paths are given as follows

$$d_2^p = \frac{\gamma - (\alpha + \lambda) - \frac{\beta - \delta}{2} D}{\mu + \frac{\beta + \delta}{2}} \quad (10)$$

$$d_1^p = d_3^p = \frac{(\mu + \beta)D^2 - (\gamma - (\alpha + \lambda))}{\mu + \frac{\beta + \delta}{2}} \quad (11)$$

This leads to the following conditions on the demand

$$d_1^p = d_3^p \geq 0 \Rightarrow D \geq \frac{\gamma - (\alpha + \lambda)}{\mu + \beta} \quad d_2^p \geq 0 \Rightarrow D \leq \frac{\gamma - (\alpha + \lambda)}{\frac{\beta - \delta}{2}} \quad (12)$$

#### 4.3. User equilibrium with information

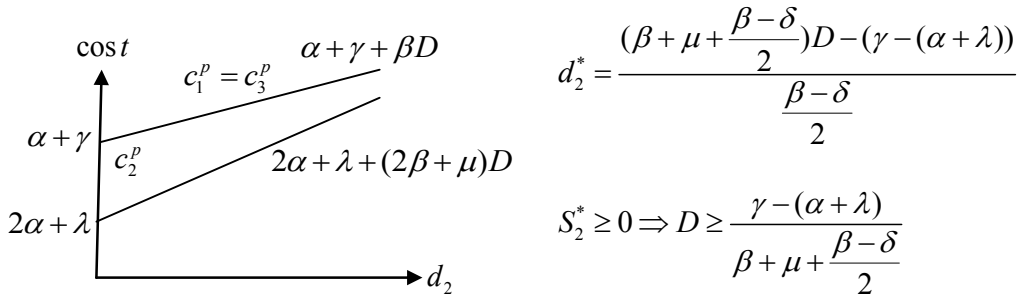
Now we calculate the user equilibrium while considering two types of users: informed users where the demand is expressed as  $D^i$  and uninformed users where the demand is expressed as  $D^n$ . The total demand is given as  $D = D^i + D^n$

The path 2 is critical for the system. So, our strategy is to ask informed travelers to avoid using this path and prefer the paths 1 and 3 instead. It will be shown that this strategy improves the cost perceived by all users in many cases

Let  $d_2^*$  designate the value of flow on path 2 such that if  $d_2 < d_2^*$ , all users have costs which are less than their costs at equilibrium. In the following, we will show that  $d_2^*$  can be determined in two of our three cases.

We study the cost function of the three cases listed above:

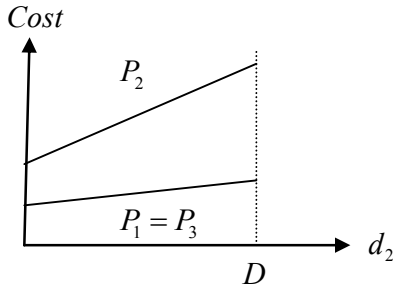
- Case 1



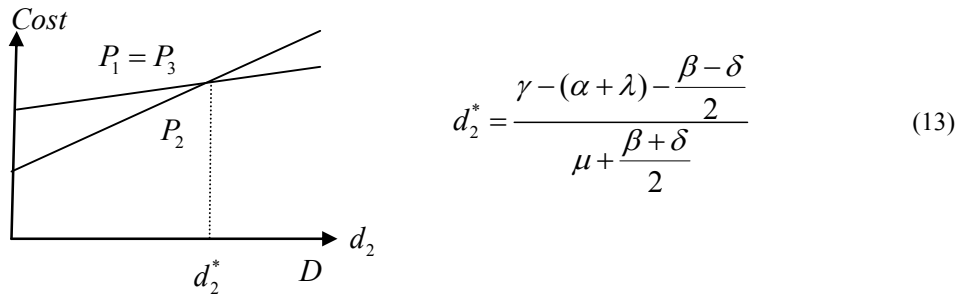
All the costs of  $d_2^p \leq d_2^*$  can be reduced by assigning  $\frac{D - d_2^*}{2}$  to each path 1 and 3 which will require to satisfy the condition  $D^i \geq D - d_2^*$ .

We improve all users' costs by assigning  $D - d_2^*$  of  $D^i$  equally on paths 1 and 3.

- Case 2: In this case, no improvement can be made because the costs will only keep on increasing



- Case 3: In this case  $d_2^*$  is the equilibrium flow given the total demand  $D$  as shown in (7)



In order to improve the travel costs, it is necessary to have  $d_2 \leq d_2^*$ , which can be obtained if,  $D^i \geq D - d_2^*$  and we assign at least a fraction  $D - d_2^*$  to paths 1 and 3

## 5. Multiagent based simulation model

In order to assess the importance of the guidance information given to travelers, simulations have been executed using a multiagent architecture constituted of two types of agents: user agents and central agents. Those agents are described as follows

### 5.1. Multiagent architecture

#### 5.1.1. Central agent

The central agent is created for each intersection and calculates the shortest path for the users using a Dijkstra algorithm that evaluates not only the cost of the arcs but also the remaining capacity on them. Once he receives the requests from the users, he creates groups of users depending on their OD pair and on the preferred modes. Three cases are given to the central agent in order to calculate the shortest path

- Case1: The user is open to taking any transportation mode  
In this case, the central agent calculates the shortest path using the Dijkstra algorithm on the multimodal graph.
- Case2: The user excludes at least one mode  
The central agent removes the arcs corresponding to that mode from the network than calculates the shortest path on the remaining graph.

#### 5.1.2. User agents

We distinguish two types of user agents: informed agents and uninformed agents. The informed agents send a travel request to the central agent with a message containing the OD pair and an ordered vector containing the preferred travelling modes. Once the agent gets a response the agent moves along the network following a cellular automata model that we will describe later. The uninformed agents, they move on the network on predefined paths.

Figure 2 describes the multiagent architecture.

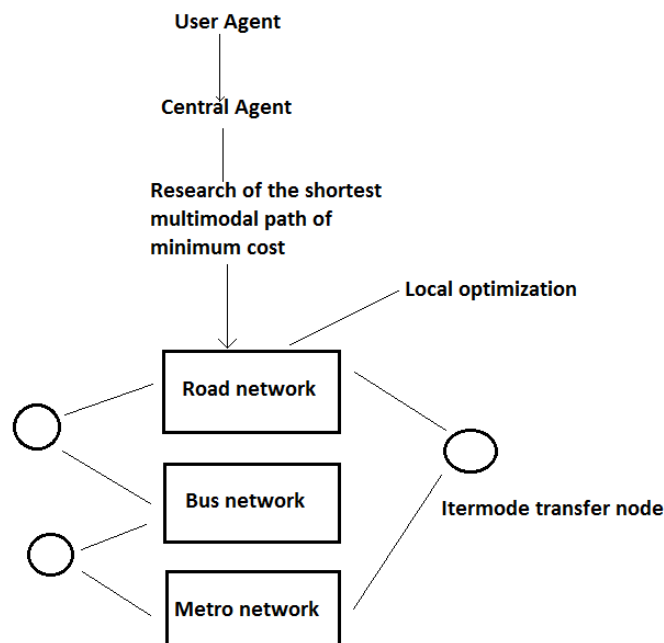


Figure 2. Multiagent architecture

### 5.2. Cellular automata model

At the entrance of an intersection, informed agents send a travelling request to the central agent along with their preferred transportation modes. At time  $t=0$ , the central agent applies a Dijkstra algorithm to find the shortest path using the free travel times then assigns the users not to the path but only to the arc. This way the shortest path is reevaluated at each intersection. After the assignment of the agent on a given arc, the travel cost on that arc is updated as the experienced travel time by the user. The movement of users on the network is given as follows:

The user checks if the first cell of the arc is free and occupies it then verifies if the next one is free, if so he moves along until the end of the arc. It is at this moment that the costs are updated as the traveling time experienced by the user. If the cell is occupied, the user waits in a buffer and renews his demand at the following simulation step. To move from one arc to another, the vehicle is buffered in a waiting queue until the first cell of the following arc is free, then he moves as described previously. The reader must also note that a buffer is created for each exit arc of an intersection but only one buffer is created at the entrance of the intersection.

When a user is assigned to a vehicular arc, the movement works differently than if it was assigned to a public transportation arc

To attend a public transportation arcs, the user must take a walking arc where he moves cell by cell. When the user attends the metro or bus node, he waits for the passage of the vehicle in an unlimited capacity buffer created at that node. The loading of the vehicles is made using a FIFO rule until the capacity of the vehicle is reached. As for the remaining users, they will wait for the following metro/bus.

The size of each cell is 10m. A bus and a metro are represented as a line of particles that move by group.

The injection of the vehicles on the network is fixed to 2 minutes for the metro and 10 minutes for the bus. The vehicles move on the network using the cellular automata described earlier. The travelling times of the vehicles are updated as the travelling time experienced by the latest vehicle.

### 5.3. Computational study

In this section, we present the computational results of our agent based simulation model for the dynamic multimodal assignment problem while differentiating between equipped and unequipped users. There are totally 22 nodes and 30 links for the multilevel multimodal network. The speed for metro is set as 20.0 m/s and for the bus and car it is set to 12.5 m/s. The stop times at stations is set to 20 seconds for both metro and bus. The vehicle capacity for metro and bus is set as 200 and 40 passengers/vehicle, respectively. The frequency of metro and bus is set as 30 and 6 vehicles per hour, respectively. As for The network under which the simulations have been executed contains three origins (nodes 22, 11 and 15) and one destination (node 10) and is represented as follow

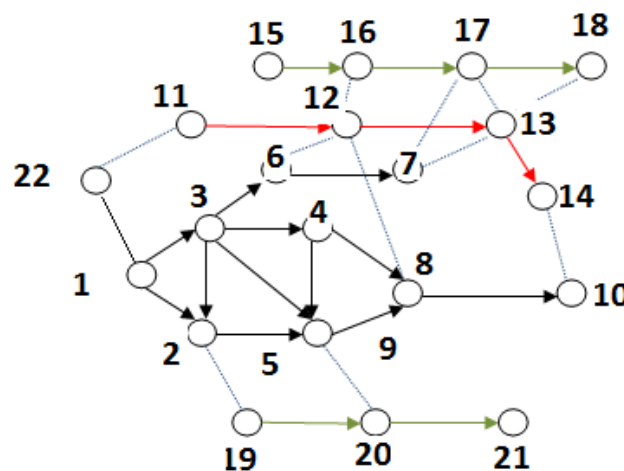


Figure 3. Multimodal graph



Note that the combination of path 1 (22-11-12-13-14-10) of cost 190 s, path 2 (22-1-2-5-9-10) of cost 220 s and path 3 (22-11-12-8-10) of cost 140 has the same structure with the Braess paradox where the critical arcs are (22,11), (11,12) and (8,10).

In order to validate the analytical results given on the importance of the information in a graph representing a Braess paradox, we are going to compare the evolution of the costs of the paths 1, 2 and 3 between two scenarios. In the first one all the users that we consider will be unequipped and will follow predefined paths and in the second scenario, we consider 50% of equipped users and 50% of unequipped users. The results are illustrated in figures (4-6).

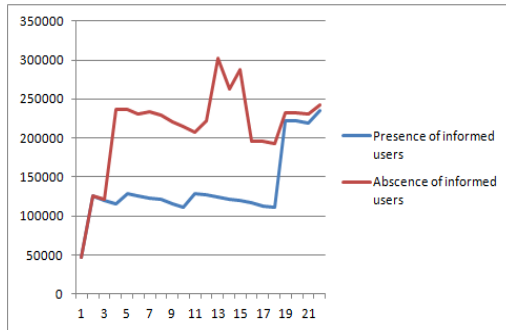


Figure 4. Comparison of the the costs between Informed and uninformed users in Path 1.

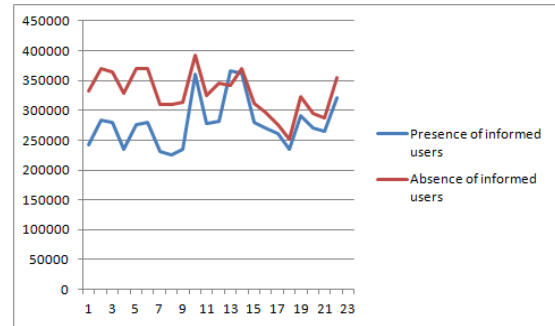


Figure 5. Comparison of between Informed and uninformed users in Path 2.

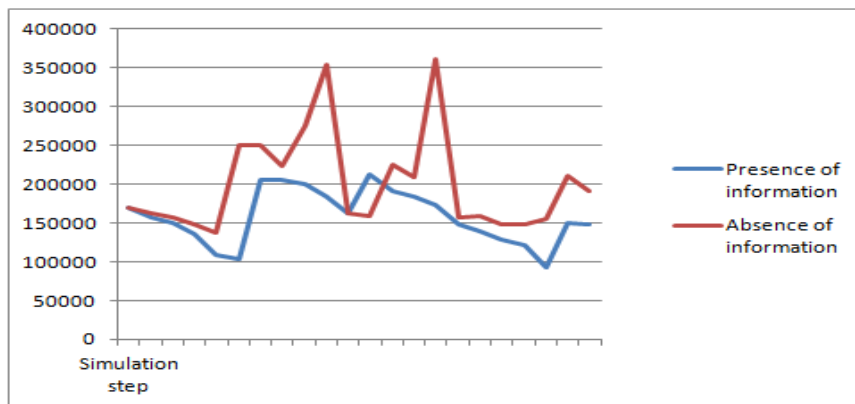


Figure 6. Comparison of the costs between informed and uninformed users in path 3

Globally, we see that the costs are lower when there is a portion of users that is guided which confirms the analytical results.

## 6. Conclusion

In this paper we have studied the influence of the guidance information that we provide the travellers on a multimodal network. We started an analytical study on a graph that describes the Braess paradox which we validated with the help of a multiagent simulation based model. In the simulations, we compared the costs on the paths when we consider only unequipped travellers with the case where we introduce a proportion of 50% of equipped users. The results were that the costs, are in general, lower when there is a proportion of guided users.

Further study includes a continuation of the work of [14] where we will try to detect the Braess paradox in order to improve the costs of all users using guidance information. An analytical study on a more general graph but also numerical experiments on a more realistic multimodal network will also be held.

## References

- Mark E.T. Horn. (2002). An extended model and procedural framework for planning multi-modal passenger journeys. *Transportation research part B*.
- G. Bellei, G. Gentile, N. Papola. (2001). Network pricing optimization in multi-user and multimodal context with elastic demand. *Transportation research part B*.
- S. Samaranayake, S. Blandin, A. Bayen. (2011). A tractable class of Algorithms for reliable routing in stochastic networks. *Procedia social and behavioral sciences*.
- Y. Hamdouch, S. Lawphongpanich. (2007). Schedule-based transit assignment model with travel strategies and capacity constraints. *Transportation research part B*.
- Y. Hamdouch, P. Marcotte, S. Nguyen. (2004). A strategic model for dynamic traffic assignment. *Network and spatial economics*.
- M. Kuwahara, T. Akamatsu. (1996). Decomposition of the reactive dynamic assignments with queues for a many-to-many origin-destination pattern. *Pergamon*.
- Hong K. Lo, C.W. Yip, K.H. Wan. (2001). Modelling transfers and non-linear fare structure in multi-modal network. *Transportation Research Part B*.
- A. Lozano, G. Storchi. (1999). Shortest viable path algorithm in multimodal networks. *Transportation Research Part A*.
- Q.T. Nguyen, A. Bouju, P. Estrailier. (2012). Multi-agent architecture with space-time components for the simulation of urban transportation systems}. 15th meeting of the Euro Working Group on Transportation.
- M. A. Kamoun. (2007). Conception d'un système d'information pour l'aide au déplacement multimodal: Une approche multi-agents pour la recherche et la composition des itinéraires en ligne. PHD thesis, Université des Sciences et Technologies de Lille et l'Ecole Centrale de Lille.
- K. Zidi. (2006). Système Interactif d'Aide au Déplacement Multimodal (SIADM). PHD thesis, Université des Sciences et Technologies de Lille et l'Ecole Centrale de Lille.
- H. Zgaya. (2007). Conception et optimisation distribuée d'un système d'information d'aide à la mobilité urbaine : Une approche multi-agent pour la recherche et la composition des services liés au transport. Ecole centrale de Lille.
- T-Y. Ma, J-P. Lebacque. (2012) Dynamic System optimal routing in multimodal transit network. *TRB*
- Y. Askoura, J.P. Lebacque, H. Haj-Salem. (2010) Optimal sub-networks in traffic assignment problem and the Braess paradox. *Computers and industrial engineering*
- M. Bruglieri, A. Coloni, A. Luè, (2014). The vehicle relocation problem for the one-way electric vehicle sharing: an application to the Milan case. *Procedia Social and Behavioral Sciences* 111, 18-27.